Q.P. Code: 19HS0830 **R19**

Reg. No:

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR

(AUTONOMOUS)

B.Tech I Year I Semester Supplementary Examinations December-2021

ALGEBRA AND CALCULUS

(Common to All)

Time: 3 hours Max. Marks: 60

(Answer all Five Units **5 x 12 = 60** Marks)

UNIT-I

1	a	Transform the matrix $A=\begin{bmatrix}-2 & -1 & -3 & -1\\ 1 & 2 & 3 & -1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & -1\end{bmatrix}$ into Echelon form and hence find its rank?	L1	5M
	b	Find the inverse of the matrix $A=\begin{bmatrix}1 & -2 & 2\\ 1 & 2 & 3\\ 0 & -1 & 2\end{bmatrix}$ using the Cayley-Hamilton theorem.	L2	7M

OR

2	a	Find the solution for the following linear system, if it is consistent. $x_1+x_2+x_3=1,\ x_1+2x_2+3x_3=6,\ x_1+3x_2+4x_3=6$.	L2	6M
	b	Reduce the quadratic form $2x^2+2y^2+2z^2-2xy+2xz-2yz$ into the canonical form and discuss its nature.	L3	6M

UNIT-II

3	a	Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e 1.1$ correct to 4 decimal places using Taylor's series.	L3	6M
	b	State and verify Rolle's theorem for the function $f(x)=\log\left[\frac{x^2+ab}{x(a+b)}\right]$ in $[a,b], x\neq 0$.	L2	6M

OR

4	a	Prove that $\frac{\pi}{3}-\frac{1}{5\sqrt{3}}>\cos^{-1}\left(\frac{3}{5}\right)>\frac{\pi}{3}-\frac{1}{8}$, using Lagrange's mean value theorem.	L1	8M
	b	Obtain the Maclaurin's series expression for the function $\cos 2x$	L2	4M

UNIT-III

5	a	Let $u=2x-y+3z,\ v=2x-y-z,\ 2x-y+z$. Verify whether u, v and w are functionally dependent? If so, find the relation.	L3	6M
	b	Find a point on the plane $3x+2y+z-12=0$, which is nearest to the origin.	L1	6M

OR

6	a	If $u = x^2 + y^2 + z^2$, where $x = e^{2t}, y = e^{2t}\cos 3t, z = e^{2t}\sin 3t$, then find $\frac{du}{dt}$.	L2	6M
	b	Examine the function $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$, for extreme values.	L1	6M

UNIT-IV

7	a	Evaluate the integral $\iint (x^2 + y^2)dxdy$ in the positive quadrant for which $x + y \le 1$.	L2	6M
	b	Evaluate the integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2 + y^2}\, dydx$ by changing into polar coordinates.	L3	6M

OR

8	a	Evaluate the integral $\int_{-1}^{1} \int_0^z \int_{x-z}^{x+z} (x + y + z)\, dydxdz$.	L1	6M
	b	Evaluate the integral $\int_0^1 \int_0^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2 + y^2}} dydx$ by changing the order of integration.	L3	6M

UNIT-V

9	a	Show that $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{\pi}{4}$.	L2	8M
	b	Express the integral $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ in terms of beta function.	L3	4M

OR

10	a	Evaluate $\int_0^1 x^4 \left(\log \frac{1}{x}\right)^3 dx$	L2	6M
	b	Prove that $\int_0^{\pi/2} \sin^2\theta \cos^4\theta\, d\theta = \frac{\pi}{32}$	L3	6M

*** END ***